THE SURVIVAL OF BABYLONIAN METHODS IN THE EXACT SCIENCES OF ANTIQUITY AND MIDDLE AGES

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> Non omnis sapientia penes Chaldaeos et Orientem fuit. Etiam Occidentis aut Septentrionis homines fuerunt λογικὰ ζῷα.
> *Scaliger,* De emend. temp. (1629) p. 171.

Among the many parallels between our own times and the Roman imperial period could be mentioned the readiness to ascribe to the "Chaldeans" discoveries whenever their actual origin was no longer known. The basis for such assignments is usually the same: ignorance of the original cuneiform sources, excusable in antiquity but less so in modern times. Given this situation, it seems to me equally important to establish what we can say today about knowledge which the Babylonians *did* have and to distinguish this clearly from methods and procedures which they did *not* have. In other words, it seems to me that it is high time that an effort is made to eliminate historical clichés, both for the Mesopotamian civilizations and their heirs, and to apply common sense to the fragmentary but solid information obtained from the study of the original sources during the last hundred years.

My approach to these problems will earn the displeasure of many scholars. Classicists who are still fighting the Persian Wars and see only barbarians in the Orientals, scholars who discover Iranian influences wherever they look, Orientalists who are convinced that "ex oriente lux," and philosophers who think that science originates from preconceived doctrines will join in disagreeing with every one of my conclusions, however different their points of view may be in all other respects. All that I can say in my defense is expressed in a sentence by Louis de la Vallée-Poussin: "je ne suis qu'un lecteur de textes."

How strongly historical interpretations can be influenced by generally accepted clichés may be illustrated by the following incident. During the excavations by the University of Pennsylvania at Nippur (1887 to 1900) for the first time a substantial number of mathematical texts came to light. Except for a few simple ideograms, they contain nothing but columns of numbers, written in the sexagesimal system. Obviously, concluded Hilprecht in editing these texts, one had here the oriental source on which ultimately rested Plato's number mysticism, contained in the *Republic* VIII and in the *Laws* V, since Plato was following Pythagoras who "derived his mathematical science and doctrines from the East." Thus Hilprecht discovered Plato's "Nuptial Number" 12960000 $= 60^4$ in his tablets which he then transcribed, e.g., as

4 3240000
5 2592000
6 . . . . . . 2160000.

In fact the numbers in the right-hand column all contain the factor $216000 = 60^3$, arbitrarily chosen by Hilprecht, while the text actually reads

4	15
5	12
6	10.

Since the product of left and right is always 60, one has here a simple table of reciprocals, telling us that ¼ of any unit is 15 minutes, ⅕ equals 12 minutes, etc.—and so with all the remaining texts which are nothing but elementary aids for sexagesimal computing.

Since 1929, when W. Struwe and I succeeded in bringing sense also into nontrivial (i.e., algebraic and geometric) mathematical cuneiform texts, our available sources have expanded to constitute probably the largest body of scientific original documents from a pre-Hellenistic civilization. No trace of number mysticism has ever been found in these often highly sophisticated but perfectly rational mathematical texts which range from the twentieth to the first century B.C. Nevertheless, the "Babylonian" origin of whatever is irrational or mystical (in fact or reconstructed) remains the inexhaustible resource for synthetic histories of science and philosophy, whether it concerns the Pythagoreans or the Ionians, Plato or Eudoxus, Nicomachus, Proclus, etc., etc.

This tendency is by no means new; in fact it is inherited from antiquity. The Babylonians or the Egyptians (e.g., in the Hermetic literature) were held responsible whenever one needed authority with a high reputation. For less inspiring subjects, like mathematics, simple nursery stories (e.g., the origin of geometry from the inundation of the fields in Egypt) gained general currency because of their simple finality. It is very illuminating to collect these ancient stories about "origins"[1] from Herodotus to the Church Fathers. Their uniformity and simplemindedness is most impressive; for our knowledge of pre-Hellenistic science these sources are not only practically valueless but seriously misleading. In order not to remain in the realm of generalities, let me quote only one example, the statements from antiquity concerning the reckoning of the beginning of the "day" in Mesopotamia. Relying exclusively on sources from classical antiquity—e.g., Pliny who claims (N.H. II, 188) that the Babylonians reckon their days from sunrise—modern scholars have come to the most contradictory results, whereas the astronomical cuneiform texts from the Seleucid period show in their date columns that sunset represents the civil epoch but that midnight was used for astronomical reasons in the computations of the lunar theory of the most advanced type ("System B"). This shows how little of contemporary Babylonian science had become common knowledge. I should not wish to extrapolate this experience to other areas of ancient cultural history and literary criticism where we do not have at our disposal original sources of unquestionable precision and authority.

That the Babylonian priest Berossos, dedicating his "Babyloniaca" to Antiochus I, transmitted Babylonian astronomy to his Greek pupils in Kos is common knowledge. Schnabel, who edited the fragments of the *Babyloniaca*,[2] added to his edition gratuitously (and with many errors) sections from two ephemerides from cuneiform texts which have nothing to do with Berossos. The little, however, that is preserved of astronomical character in the fragments[3] suffices to demonstrate that Berossos was totally ignorant of the contemporary Babylonian astronomy when he was teaching that the lunar phases were the result of a rotation of the moon which he supposed to be half luminous, half dark. The mathematical theory of the lunar phases constitutes the best developed and most sophisticated section of Babylonian astronomy in the Seleucid period, leaving no room for such primitive doctrines. They were proper meat for Greek philosophers; for the transmission of Babylonian astronomy, however, Berossos can be safely ignored.

If we now turn to positive evidence, provided by cuneiform texts, we stand on safe chronological grounds. Mathematics is fully developed in the Old Babylonian period, while mathematical astronomy originated probably in the Persian period. Cuneiform texts of both classes still exist from the last century or two B.C. Of the origin of Babylonian mathematics, we know nothing; my guess would be that it developed fairly rapidly without a Sumerian antecedent. Whatever the case may be, we know today that it had reached by the nineteenth century B.C. a full command of sexagesimal techniques based on a place value notation (though without a symbol for zero), including higher exponents and their inverses, and a great deal of insight into algebraic and plane geometric relations, among which "Thales' Theorem" about the right triangle in a semicircle and in particular the "Pythagorean Theorem" for the right triangle take a common place. The famous Plimpton Tablet[4] reveals full understanding of the mathematical laws which govern "Pythagorean" triples of integers, i.e., solutions of $a^2 + b^2 = c^2$ under the condition that a, b, and c be integers. To quote only the first three solutions on which our text is based:

a	b	c
2,0 (= 120)	1,59 (= 119)	2,49 (= 169)
57,36 (= 3456)	56,7 (= 3367)	3,12,1 (= 11521)
1,20,0 (= 75600)	1,16,41 (= 4601)	1,50,49 (= 3949) .

This goes far beyond such trivialities as the "discovery" that $3^2 + 4^2 = 5^2$.

Since we have mathematical cuneiform texts from the Seleucid period and since Greek and Demotic papyri from the Greco-Roman period in Egypt show knowledge of essentially the same

[1] *Cf.* for this whole problem A. Kleingünther, ΠΡΩΤΟΣ ΕΥΡΕΤΗΣ, *Philologus,* Suppl. **26,** 1 (1933).

[2] *Berossos und die babylonisch-hellenistische Literatur* (Leipzig, 1923).

[3] Schnabel, Nos. 16 to 26.

[4] O. Neugebauer and A. Sachs, *Mathematical Cuneiform Texts* (American Oriental Series **29**, New Haven, 1945), pp. 38–41.

basic material, one can no longer doubt that the discoveries of the Old Babylonian period had long since become common mathematical knowledge all over the ancient Near East. The whole tradition of mathematical works under the authorship of Heron (first century A.D.), Diophantus (date unknown), down to the beginning Islamic science (al-Kwârazmî, ninth century) is part of the same stream which has its ultimate sources in Babylonia.

Probably also in the Persian period, perhaps in connection with beginning mathematical astronomy, the sexagesimal place value notation was perfected by the introduction of a symbol for "zero" (a separation mark). Thus, when Greek astronomy began its own development in the early second century B.C., using sexagesimally written Babylonian parameters, the use of a separation mark for zero was also adopted. With Greek astronomy the place value notation, including zero, came to India, where this system was finally extended also to decimally written numbers, whence our "Hindu (Arabic) numerals" originated. Again, the basic idea is undoubtedly Babylonian in origin.

For Greek mathematics the picture now becomes quite clear. It hardly needs emphasis that one can forget about Pythagoras and his carefully kept secret discoveries. It is also clear that a large part of the basic geometrical, algebraic, and arithmetical knowledge collected in Euclid's *Elements* had been known for a millennium and more. But a fundamentally new aspect was added to this material, namely the idea of general mathematical proof. It is only then that mathematics in the modern sense came into existence. Parallel with the development of modern axiomatic mathematics it became clear that the discovery of "irrationals" by Theaetetus and Eudoxus caused the transformation of intuitively evident arithmetical and algebraic relations into a strictly logical geometric system. From this moment the theoretical branch of Greek mathematics severed all relations with the ultimately Mesopotamian origins of mathematical knowledge.

It is very illuminating to compare Euclid's *Elements* with another work by the same author, his spherical astronomy (the *Phaenomena*).[4a] In the *Elements* everything is subject to a perfectly logical structure (though notoriously based on different earlier presentations, in part of the fifth century). In the *Phaenomena* we find only fumbling attempts to obtain some quantitative and some geometric insight into the mutual relations of equator, ecliptic, and parallel circles in relation to the yearly solar motion. The *Elements* were unsurpassed for two millennia to come; the *Phaenomena* try to settle elementary astronomical questions which had to wait for four more centuries of efforts until Menelaos' spherical geometry. I think the reason for such a marked difference is clear. In the *Elements* a well-known body of material was readily at hand and made subject to the newly discovered logical principles—and it is worth repeating that this constitutes a scientific achievement of the very first order but adds very little to the factual material. For spherical geometry, however, the situation was quite different. All that we know of Babylonian astronomy (and mathematics) speaks against the existence of any spherical geometry. Crude arithmetical schemes mark the beginning of astronomical endeavors—e.g., concerning the variation of the length of daylight during the year—eventually to be highly refined by means of arithmetical sequences of different order. Here the Greeks had to begin from scratch. Eudoxus' "Homocentric Spheres" were a flash of genius but astronomically valueless, and the subsequent entanglement with philosophical principles did not help matters. Euclid and Aristarchus drastically demonstrate the inadequacy of the traditional mathematics to cope with spherical astronomy and trigonometry at the end of the fourth century. Only with Apollonius (around the end of the third century) does real progress begin by returning to plane geometry, masterfully applied in Archimedean fashion to the problems of eccentric and epicyclic motion. We do not know whether Apollonius had Babylonian data at his disposal; from what we know about his work, there is no compelling reason to assume it. With his successor Hipparchus, however, it is clear that the empirical data of the contemporary Babylonian astronomy were available to him while in all probability the theory of stereographic projection (perhaps influenced by Apollonius' theory of conic sections) made it possible to solve spherical trigonometric problems in the plane. From now on, the full impact of the Babylonian sexagesimal place value notation is felt and remains the backbone of astronomical

[4a] Unfortunately I did not overlook a recent paper on "Greek Astronomy and its Debt to the Babylonians" by Leonard W. Clarke in *The British Journal for the History of Science* **1** (1962): 65–77. May it suffice to characterize the author's competence by his opinion "that Euclidean geometry pre-supposes a flat earth" (p. 75).

computations to the present day such that the sexagesimal division of time and angles still encumbers instruments and tables. Only in the first century of our era did Menelaos find the proper generalization of plane geometry to the sphere by operating with great circles only and thus create spherical trigonometry. With Ptolemy, in the next generation, astronomy reaches the same perfection as mathematics in the *Elements* almost five centuries earlier. He eliminated the remnants of Babylonian-Hipparchian parameters by the systematic refinement of all empirical data. This was made possible by comparison of his own observations and improved cinematical models with older data. Except for the sexagesimal computational procedures the role of Babylonian astronomy is now ended for mathematical astronomy.[5] Exactly as Babylonian mathematics lingers on for many more centuries in the Heronic-Diophantine literature which is continued into the high Middle Ages, so too we shall find residues of Babylonian astronomy in branches untouched by the Almagest tradition, in particular in the Hindu-Arabic tables and treatises. But for the scientific main stream, the Babylonian influence ends for Greek mathematics in the fifth century B.C. and for astronomy with Menelaos and Ptolemy in the first and second century A.D.

Astral religion has as little to do with the origin of astronomy as Genesis with astrophysics. Nor does the interest in celestial omens—as one class of omens among many—lead to astronomy. As long as one takes extraordinary events as a means of communications between the gods and men one will devote every effort to deciphering this complicated divine language. One will record the repetitions of phenomena in order to find their significance but one will not try to predict the recurrence of the ominous phenomena themselves. Astrology, which operates with the use of mathematical astronomy, is the very antithesis of omen-lore. Its basis is causality, not communication of arbitrary decisions of a divine will. This causal root of astrology is still clearly recognizable in Greek astrology in its relation to weather prognostication. Just as sun and moon visibly influence the seasons, the winds, and climatic conditions, so may also the stars and planets be influential. Now it makes sense to predict; but Hesiod and Aratus, the parapegmata and related treatises, show how qualitative and how crude all such attempts had to remain before the necessary mathematical tools existed. The situation in Mesopotamia was perhaps not very much different. It is probably not accidental that the first calendaric cycles, in particular the "Metonic" nineteen-year cycle, appear simultaneously in Babylon and in Greece. In both areas simple arithmetical schemes gained wide currency: in Babylonia the schemes for the variable length of daylight and night, in Greece similar schemes for the determination of hours by means of the length of the human shadow. Very little is known of an "astrology" in the Hellenistic sense of the word from this early time in Mesopotamia. But the really decisive difference lies in the fact that the Babylonians made a serious—and in the end extremely successful—attempt to control their lunar calendar mathematically by unravelling step by step the different periodic variations which cause the intricate pattern in the variation of the time between consecutive new crescents. The resulting mathematical procedures represent undoubtedly one of the most outstanding scientific achievements of antiquity. Its direct effect was, however, very small indeed as its scientific content is concerned; but peripheral elements had an enormous spread, geographically as well as chronologically, and provide for the historian who is familiar with the original context one of the most powerful tools for establishing the routes of transmission of scientific methods.

The popular belief of antiquity that astrology originated with the "Chaldeans" certainly contains an element of truth, although the textual evidence for Babylonian astrology (as distinct from celestial omens) is rather meager and belongs only to the latest periods. This fact has caused W. Gundel to go to the other extreme and to seek the origin of astrology in Egyptian civilization. In fact, however, one can be sure on the basis of all Egyptian documentation that the only component of ultimately Egyptian origin in astrology is the "decans" which were assimilated to the zodiac of Babylonian origin in the early Ptolemaic period. But the enormous expansion of astrology into an all-encompassing doctrine is undoubtedly a purely

[5] As another example of historical clichés in which practically every sentence is wrong may be quoted (remarks in [ ] are mine): "The final synthesis by Ptolemy in the Almagest . . . shows in fact great mathematical resourcefulness, a new use of Babylonian techniques, but no change except a yielding of principles: the uniformity of circular motion . . . had to be abandoned at last [why?] in favor of more complicated hypotheses. But by that time no one thought of forcing his way back [to where?]. The tool [which one?] controlled the men." (Giorgio de Santillana, *The Origin of Scientific Thought* (Chicago, 1961), p. 250).

Hellenistic product, developed from a comparatively modest Babylonian nucleus.[5a] It is perhaps useful occasionally to remember that the so-called Greek mind not only produced works of the highest artistic and intellectual level but also could indulge in the development of the most absurd doctrines of a pseudo-rational superstition which contributed heavily to the "darkness" of later ages.

For the spread of certain scientific methods, however elementary, the ultimate Babylonian origin of astrology is of great importance. Most of the methods which spread with Hellenistic astrology to India and (directly or indirectly) to the West belong to the elementary level of Babylonian astronomy. Thus early Babylonian-Hellenistic scientific methodology, at that level, remained the main tool of astrological practice and the refinements neither of Babylonian mathematical astronomy nor of the Almagest had an essential influence on the formation of astrological methodology for the determination of the position of the celestial bodies and for questions of spherical astronomy. There are strong indications, however, that much which we find of Hellenistic material in Hindu astronomy reflects the situation of astronomical knowledge in the time of Hipparchus. That direct Babylonian influence reached India seems to me not very likely—e.g., all the foreign technical terms are Greek—such that Babylonian components in Hindu astronomy can be taken as evidence for a corresponding influence on Hellenistic astronomy for the early period. This view is supported by the discovery, in Greek papyri of the early Roman imperial period, of methods which are characteristic for a certain class of sources from India (Tamil).[5b] Only in passing may be remarked that there is no evidence whatever for a Babylonian origin of the concept of "lunar mansions," however frequent such an origin has been assumed in the literature.

Beside the use of the sexagesimal number system and the zodiacal division of the ecliptic, the employment of the "lunar days" of exactly $\frac{1}{30}$ of a mean synodic month goes back to Babylonian astronomy. The consistent use of a strict lunar calendar made it necessary to introduce smaller units in terms of the lunar month as fundamental unit of time measurement. This clear and convenient definition was perverted in later Indian astronomy and astrology to the use of thirtieths (*tithi*) of true lunar months, i.e., to units variable in a very complicated fashion from month to month. In other words, the Hindus reintroduced into the definition of *tithis* (as they are still used in India today) exactly the complication which it was the purpose of the Babylonian invention to avoid.

Since from early times the Babylonian calendar (not the Assyrian one) had the tendency to coordinate the lunar calendar more or less with the seasons of the solar year, the mathematization of astronomy in the fifth century B.C. is also reflected in a definite intercalation cycle, usually called the "Methonic cycle," which intercalates seven additional months in nineteen lunar years. This quite accurate and convenient cycle in combination with the continued counting of the regnal years of Seleucus I, beginning at 312 B.C., constitutes one of the greatest advances in practical chronology. Here we have for the first time a precise era in which dates can be accurately established according to simple computational rules. It is not surprising that Islamic astronomers made much use of this era (or a modification, the era of Philip) and it should be mentioned that the chronology of the modern historians for the Hellenistic age is based on Father Epping's decipherment of the terminology of astronomical cuneiform texts in their relation to the Seleucid era. Another aspect of the later history of the nineteen-year cycle is contained in the stormy history of the Easter cycles—analogous to the history of the *tithis* in so far as a simple and practical solution of one problem was contaminated by additional conditions (e.g., Easter limits) which deprived the original solution of its main value, simplicity. Indeed, that simplicity was the element that recommended the nineteen-year cycle to the Babylonian astronomers is demonstrated by the fact that the mathematical astronomical texts of the whole Seleucid-Parthian period maintained the use of the nineteen-year cycle as the chronological skeleton of their computations in spite of the fact that they used, for their lunar ephemerides themselves, relations of higher accuracy than those reflected in the calendaric cycle.

That much of the astronomical elementary

[5a] For clear evidence of Babylonian astrology in Hellenistic Egyptian texts *see* R. A. Parker, *A Vienna Demotic Papyrus on Eclipse- and Lunar-Omina* (Providence, Brown University Press, 1959).

[5b] The whole problem of Babylonian influences on astronomy and astrology in India is now discussed in a masterful article by David Pingree, *Isis* **54** (1963): 229–246.

knowledge of later times originated in Babylonian astronomy is not difficult to show. For example relations for planetary phenomena, fundamental for the Babylonian "Goal-year-texts" [6] (but refined in the mathematical-astronomical texts), reappear in Greek and mediaeval astrological treatises. Similarly, certain patterns for the anomalous motion of the moon and of the planets have been found in Greek as well as in Demotic papyri [7] while their use in Indian and in Islamic sources is equally attested.[8]

A particular modification of an early scheme in Babylonian astronomy has greatly influenced ancient geography: I refer to their arithmetical patterns for relating the variable length of daylight to the position of the sun in the ecliptic. This simple scheme (existing in two variants, "System A," strictly linear, and "System B," with double the ordinary difference in the middle of the increasing and decreasing branches) was adopted, probably in the second century B.C.,[9] to the latitude of Alexandria and subsequently to other geographical latitudes. There is no trace anywhere in Babylonian astronomy for the concept of "geographical latitude" and consequently for provisions necessary for the adaptation of their procedures for other localities. Had astronomy originated during the Assyrian empire the situation might have been different; but the astronomers of the fourth century could not feel the need to see their computation applied outside a narrow area from Uruk to Babylon.

The world of the Greeks in Alexandria was of different dimensions; it extended from the Far East and India to Spain and from the Upper Nile to the Crimea and beyond. Obviously the Babylonian scheme for the variation of the length of days and nights would not do for such an area. But the mathematical device in itself was simple and easy to modify—again in a typical Babylonian fashion by a linear variation of the extremal length of daylight but otherwise unchanged pattern. According to this scheme one distinguishes "climates" of equal length of daylight, arranged in the simple pattern of half-hour increment of the longest day. Again, as everywhere else, the strictly Babylonian procedure was eventually eliminated (probably shortly before Ptolemy, if not by himself) when Greek spherical trigonometry replaced the cruder arithmetical patterns. But, as a concept, the sequence of the climates of linearly increasing length of the longest day remained unchanged and dominated geographical lore from antiquity through Islam and the western Middle Ages. Simultaneously the original Babylonian scheme (even in such details as the definition of the eighth degree of Aries as the solar position at the vernal equinox) remained in use in the astrological literature (e.g., among many others, in the *Anthology* of Vettius Valens of the second century A.D.) and spread with it to India, where we find the unchanged Babylonian System A in the writings of Varaha Mihira (sixth century A.D.) applied for latitudes entirely different from Babylon. Intelligent modifications of the Babylonian arithmetical scheme for the latitude of Persia are described by al-Bîrûnî (around A.D. 1000) as used by "the people of Babylon."[10]

The absence of recognition in Babylonian astronomy of any influence of geographical coordinates on astronomical procedures makes it obvious that at no time of its existence could Babylonian astronomy predict that the path of a certain solar eclipse—whether visible at Babylon or not—would cross Asia Minor. In other words it is clear that even the methods of the Seleucid period would not explain the alleged approximate prediction by Thales of a solar eclipse for Ionia. Since for a given region no simple periodicity of solar eclipses exists, the only way to predict a solar eclipse would have been to investigate mathematically every conjunction (or at least every sixth conjunction) for the given place and moment. This is indeed the procedure still followed by Ptolemy and by Islamic and Byzantine astronomers—not before the Renaissance could one compute eclipse paths, because this requires a much better knowledge of the solar parallax than the observational methods of antiquity, restricted to naked-eye techniques, could provide. That Thales had even the faintest idea of the problems involved is out of the question, quite aside from the fact that Mesopotamian "astronomy" of the early sixth century B.C. made at best the first stumbling attempts to construct quantitative schemes to describe the variation of

[6] *Cf.*, for this concept, A. Sachs, "A Classification of Babylonian Astronomical Tablets of the Seleucid Period," *Journal of Cuneiform Studies* **2** (1950) : 271–290.

[7] E.g., R. A. Parker, "Two Demotic Astronomical Papyri in the Carlsberg Collection," *Acta Orientalia* **26** (1962) : 143–147.

[8] I am using here unpublished material recently uncovered by E. S. Kennedy.

[9] Hypsicles' *Anaphorikos* is our earliest source.

[10] Mark Lesley, "Bîrûnî on Rising Times and Daylight Lengths," *Centaurus* **5** (1956–1958) : 138.

the length of daylight or to measure the shadow lengths.

While the enthusiasts for Ionian philosophy will not desist from allowing Thales to borrow non-existing methods from Babylonian astronomers, another fabulous achievement, the discovery of the precession of the equinoxes, has a better change of disappearing from the literature because this amounts to restoring a pearl to the crown of the Greeks. Since Schnabel's theory of the Babylonian discovery of precession was based on taking seriously a scribal error (interchange of cuneiform 4 and 7—as common as Α and Δ in Greek), it sufficed to find the other half of the text, extant in Chicago and unknown to Schnabel, containing other scribal errors more than outbalancing the first one. In fact before Newton's understanding of the relation between the precession of the equinoxes and the flattening of the earth's globe, the discovery of a slow gradual change in the longitudes of stars is neither a great achievement nor of theoretical interest for ancient astronomers—it only requires the preservation of records sufficiently old to establish beyond doubt the existence and amount of the effect.

The really significant contribution of Babylonian astronomy to Greek astronomy, in particular to Hipparchus' astronomy, lies in the establishment of very accurate values for the characteristic parameters of lunar and planetary theory [11] and in particular in the careful separation of the components of the lunar motion—longitude, anomaly, latitude, and nodal motion. The value of one of these parameters, the evaluation of the length of the mean synodic month as 29;31,50,8,20 days is not only fundamental for Hipparchus' theory of the moon but still appears in the Toledan Tables of the eleventh century. Islamic astronomy of the ninth century received its first impulses from Persia and India and assimilated the Ptolemaic refinements only somewhat later (in particular through al-Battânî, around 900). The earlier phase is mainly represented by al-Khwârizmî and influenced Spanish Islamic astronomy which remained somewhat outside the development in the Near East, Persia and Byzantium. But the European revival of astronomy beginning in the twelfth and thirteenth centuries took place mainly in Spain and Southern France and thus reflects again the Hindu, ultimately Babylonian, component of Islamic astronomy. Not until the full recovery of the Ptolemaic methods in the Renaissance of the fifteenth century did the influences directly traceable to Babylonia of the Seleucid-Parthian period disappear.

For the modern historian there remains as the greatest unsolved question the problem of the transmission of astronomical knowledge from the temple schools in Uruk and Babylon to men like Hipparchus or Apollonius. The situation for astronomy is very different from the parallel situation in mathematics. This difference is very outspoken even in modern historical research. Ancient pre-Greek mathematics is easy to understand since it concerns only the elementary facts of arithmetic, geometry, and algebra. This material must have been accessible in countless elementary treatises at all periods and in all areas of the Near East. The astronomy of the Hellenistic period is a quite different matter. Not that the mathematical methods in themselves are more advanced than in the ordinary contemporary mathematics. The real difficulty lies in the astronomical motivation for the complex interplay of difference sequences which represent the different components of lunar and planetary motions. The determination of the characteristic parameters of periodic difference sequences of different order as well as the design of interpolation methods applicable to these difference sequences requires arguments totally outside the framework of ordinary ancient mathematics—in fact often strangely similar to the numerical methods of the latest Islamic period and to the beginning of modern "applied" mathematics.

It is obvious that the transmission of this type of material cannot be ascribed to any latent knowledge contained in easily accessible treatises. Even if we completely disregard the very serious practical difficulty of utilizing cuneiform material, we must assume a careful and extended training by competent Babylonian scribes and computers in order to account for the profitable use of any of the Babylonian ephemerides. We have at our disposal enough cuneiform instructions for the computation of lunar and planetary ephemerides to be able to say that they require a great deal of study and additional instruction and astronomical knowledge before it is possible to use them properly. And of the arguments upon which this mathematical theory was constructed these "Procedure Texts" contain nothing. It is therefore not surprising that the Greek astronomical literature does not contain a trace of factual information concerning the theo-

[11] *Cf.* Asger Aaboe, "On the Babylonian Origin of some Hipparchian Parameters," *Centaurus* **4** (1955–1956): 122–125.

retical foundations of Babylonian astronomy. Perhaps this silence is not accidental but reflects the fact that Greek astronomy did not know too much about the details of the Babylonian techniques and their theoretical and historical foundations. This might have forced the Greeks to look for methods of their own to solve the problems which arise in the mathematical description of the motion of the celestial bodies, a task which Apollonius and Hipparchus began and Ptolemy completed by bringing the planetary theory to the same degree of perfection as the lunar theory of old, but on the basis of cinematic models and spherical trigonometry, both unknown to his Babylonian predecessors.